MATHEMATICS TEACHERS’ WORK WITH CURRICULUM RESOURCES
Ghislaine Gueudet

To cite this version:
Ghislaine Gueudet. MATHEMATICS TEACHERS’ WORK WITH CURRICULUM RESOURCES. ENEDIM, 2017, Athènes, Greece. <hal-01662515>

HAL Id: hal-01662515
https://hal.archives-ouvertes.fr/hal-01662515
Submitted on 13 Dec 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
MATHEMATICS TEACHERS’ WORK WITH CURRICULUM RESOURCES

Ghislaine Gueudet
CREAD, ESPE de Bretagne, University of Brest
Ghislaine.Gueudet@espe-bretagne.fr

Mathematics teachers have always worked with many curriculum resources. These resources include textbooks, computer software; they have recently evolved to incorporate a profusion of online resources, leading to important evolutions of teachers’ work. To understand and study these evolutions, we have developed a theoretical approach in mathematics education: the documentational approach to didactics. In this conference, I briefly introduce the main principles of this approach, and I present examples of research projects that used this approach to study the interactions between teachers and curriculum resources and their consequences.

CONTEXT AND RELATED WORKS

Mathematics teachers have always worked with many curriculum resources, defined here as “all the resources, which are developed and used by teachers and pupils in their interaction with mathematics in/for teaching and learning, inside and outside the classroom” (Pepin & Gueudet 2014).

Amongst these resources, the textbook plays a central role. Textbooks and their use have been the focus of many research works in mathematics education (see e.g. Pepin & Haggarty 2001; Remillard 2005; Fan, Jones, Wang & Xu 2013). Textbook studies can focus on the content of the textbooks; on their design; on their use by the students or the teachers; but also on the consequences of this use in terms of teachers’ professional development. In the context of educational reforms, textbooks are sometimes designed by the educational authorities to support a change in the teachers’ practices (Ball & Cohen, 1996). Textbooks certainly influence teachers’ practices; nevertheless, the textbook alone cannot lead to a deep modification of the teachers’ practices. Remillard (2012) has evidenced that teachers’ use of textbook must be viewed as an interaction. When using a textbook, even a new one, the teacher is guided by his/her usual “mode of engagement” with textbooks. At the same time, this “mode of engagement” has been developed along the years through the use of many textbooks,
hence it depends on the features of these textbooks. The perspective we propose on the interactions between teachers and curriculum resources is very close from the perspective developed by Remillard. Nevertheless it introduces new concepts, in order to take into account the recent evolutions of the available curriculum material.

Indeed curriculum resources include textbooks, teachers’ guides, but also manipulatives or computer software. Moreover they have recently evolved to incorporate a profusion of online resources: websites, interactive exercises, and in particular more and more Open Educational Resources. This availability of Open Educational Resources (OERs) produces drastic changes in education, and in teachers’ work in particular. This fact has been acknowledged for several years at the policy level (see e.g. OECD 2007). For example, a text of the European parliament, following a report on new technologies and open educational resources by the Committee on Culture and Education states that:

“[The European parliament] emphasises that OERs create opportunities for both individuals, such as teachers, students, pupils and learners of all ages, and educational and training institutions to teach and learn in innovative ways; calls on educational institutions to further assess the potential benefits of OERs in the respective educational systems.” (European commission 2013)

The OERs are potentially disruptive, not only for learners but also for teachers who can transform the available material, share it with colleagues etc. (Trouche, Gueudet & Pepin to appear). This potential is naturally linked with the issue of new technologies –open educational resources and digital technologies are strongly connected. Indeed the Internet is the central means that allows finding and even designing and publishing open resources; moreover, some of these resources can themselves be considered as “new technologies”: free educational software, e-textbooks etc.

For these reasons we claim that an approach aiming at researching these evolutions needs to be connected not only with textbook research, but also with research on the use of technology in mathematics education (Hoyles & Lagrange 2010). Recent works on technology use have also acknowledged the fact that understanding the use of ICT by teachers requires taking into account their resource system (Ruthven 2012). In the next section we present the approach we developed, drawing on these previous works.
THE DOCUMENTATIONAL APPROACH: THEORY AND METHOD

Central concepts of the theory

The main theoretical source of the documentational approach is the instrumental approach (Rabardel 1995) developed by Rabardel. The instrumental approach defines an artefact as a product of the human activity, designed for a human activity with a given aim. The instrumental approach itself is indeed rooted in activity theory (Vygotsky 1978), and considers subjects engaged in an activity with a given aim or object.

Along his/her activity with the artefact, the subject develops an instrument. An instrument is a mixed entity, comprising the artefact or parts of it, and a scheme of use of the artefact (Vergnaud 1998). A scheme is a stable organisation of the activity for a class of situations (a set of situations corresponding to the same aim of the activity). Two different subjects, starting from the same artefact, can develop different instruments.

The instrumental approach has been used in mathematics education to study how students learn mathematics with technology (Guin, Ruthven & Trouche 2005). For example with the artefact “graphic calculator”, two grade 10 students can develop two different instruments for the aim “studying a function”. One student can start with the graphic tools, without a preliminary choice of an appropriate window; while another can start with a table of values, in order to choose a window before trying to see the graph of the function.

The development of the instrument is a process called “instrumental genesis”. This genesis comprises two intertwined movements: on the one hand, the features of the artefact influence the schemes developed by the subject, this is called “instrumentation”. On the other hand, the subject himself/herself modifies the artefact, according to his/her personal knowledge or choices: this is called instrumentalisation.

We claim that considering not only ICT, but all the resources available for the mathematics teacher requires a concept of resources overcoming the artefacts. Such a concept of resources has been introduced by Adler (2000), who considers that a resource for a teacher can be anything “likely to resource the teacher’s practice”. An artefact can be a resource; but here the focus is on the use by the teacher. For example a teacher in preschool can use an orange (which is not an artefact) to provide a representation of a round shape. The students’ productions are also important resources for the teacher.
The main principles of the documentational approach have been introduced in Gueudet and Trouche 2009, or Gueudet, Pepin and Trouche 2012 for example. We consider that a teacher, in his/her work, interacts with a variety of resources. The teacher looks for resources, sometimes just meets resources, stores them, then transforms them, associates several resources, sets them up in class etc. We call this the teacher’s documentation work. Along this work, from a set of resources the teacher develops a document: a mixed entity, comprising the selected resources and a scheme of use of these resources. For example, a teacher can develop a document from the class textbook, for the aim “giving homework”. He/she chooses two exercises corresponding to the lesson; one easy exercise, a direct application, and one more complex and gives them has homework for the next course. This is a stable organisation of his/her activity, hence a scheme (which can be described with more details, I will not present it here).

The development of a document is a process called a documentational genesis. Alike the instrumental approach, it is composed of two intertwined processes: instrumentation and instrumentalisation (Figure 1).

Table 1: Representation of a documentational genesis (Gueudet, Pepin & Trouche 2012).

Along his/her work, the teacher develops a resource system: a structured set of resources, organised according to the different aims in his/her activity: introducing new notions, practising learned techniques, assessing, for example.
Investigating the teachers’ resources: methodology

Investigating the teachers’ resources raises important methodological questions. Teachers can discover resources in many different places and at any moment: not only at school, but also at home; discussing with their colleagues, but also with their family etc. A researcher who wants to follow in details the use of resources by a teacher should stay with this teacher night and day! A second difficulty is that geneses are long-term processes. A scheme is a stable organisation; so observing schemes requires following the teachers over long periods of time. For all these reasons, we have developed a special methodology, called “reflexive investigation”.

The main principles of this methodology are:

- A long-term follow-up of the teacher, ranging from several weeks to several years;
- A close association with the teacher, who will collect data him/herself: hence the name “reflexive investigation”. When starting to work with teachers on their resources, they are generally not aware of the variety of resources they actually use. They can say things like: “you know, I use only a few resources, always the same”. Working on the description of these resources, they become aware of their number and variety;
- A collection of all the resources used and produced;
- Observations in class, to confront the teacher’s declaration and his/her actual practice.

The data produced following these principles include interviews, classroom videos, resources used and designed by the teacher (in particular students’ productions) and different kinds of descriptions produced by the teacher: logbooks, or graphical representations of their resources. These representations, called “Schematic Representation of the Resources System” inform us about how the teacher views the organisation of his/her resources. Various treatments of these data can then be organised, according to the research question investigated.

The Documentational Approach can be used to study a large variety of questions: which evolutions of the design modes of resources? Which criteria guide the teachers’ choices of resources, the modifications of the resources they choose? How does the work with resources influence teachers’ professional development, when they work individually, or when they work collectively, for example in teacher education programs? We develop below two examples of research works using the Documentational
Approach, and investigating different kinds of questions in different contexts.

THE VIRTUAL ABACUS: A STUDY AT PRIMARY SCHOOL

Presentation of the virtual abacus

The Virtual abacus (figure 2) is an OER developed in France by Sésamath, an association of mathematics teachers producing online resources. Alike all other Sésamath resources, it is freely available online¹, and can also be downloaded to be used without Internet access. I briefly recall here the principles of the abacus.

![Virtual Abacus](image)

**Figure 2. An interactive exercise on the virtual abacus**

The Chinese abacus is separated in two parts by a central bar (the material abacus must be used horizontally, while the virtual abacus appears as vertical). The central bar is called “the reading bar”: only the beads on this bar are considered as “activated”. There are two kinds of beads: 5-unit beads (two of them, in the upper part for the virtual abacus) and 1-unit beads (five of them, in the lower part of the abacus). The Chinese abacus comprises 13 vertical rods. Each rod corresponds to a rank of the place-value system: units, tens, hundreds, etc. (from the right to the left). There are several possibilities to display the same number on the Chinese abacus: for example

---

¹ [http://cii.sesamath.net/lille/exos_boulier/boulier.swf](http://cii.sesamath.net/lille/exos_boulier/boulier.swf)
10 can be represented by two 5-unit beads on the last rod on the right (representing units), or by one 1-unit bead on the rod on its left (representing tenths).

On the virtual abacus, the beads can be moved using the mouse. Figure 2 represents an interactive Sésamath exercise, where the student is asked to display a given number. Here the student made a typical mistake: he/she used two 5-unit beads instead of two 1-unit beads.

We worked during several years within research and design groups, associating teachers, teacher educators, and researchers and designed different kinds of resources for the teachers (in particular within a French national project called “Mallette Mathématique pour l’école, financed by the ministry of education), and an online training path, around the virtual abacus (Gueudet & Bueno-Ravel 2016).

**Integration of the virtual abacus by a teacher in a grade 4 class**

Rose is a teacher at grade 4 (students aged 9 to 10 years old). In her class she has a computer for the teacher, a video projector, a visualizer, 15 laptops for the students and 12 material abaci.

We followed Rose in her class during a sequence of lessons devoted to “large numbers” (Poisard, Gueudet & Robin 2016). Rose uses clearly a resource system during this sequence. The technological resources: laptop, visualizer, virtual abaci etc. have been integrated in her usual resource system because of their possible link with the material abacus. Indeed Rose declares: “It is important for the students to touch things and to move them”. This is typically an operational invariant: a theorem-in-action, a proposition considered as true, and leading the teacher’s practice. For this reason, Rose wanted to use the abacus with her class; but it is too difficult to organise a lesson with only the material abacus, the teacher cannot observe simultaneously all the students, and the beads can move very quickly. Hence she decided to use the virtual abacus, together with the material one. She also used traditional students’ sheets on paper with the laptops, and traditional posters on paper with the video-projector and visualizer.
Rose combines the traditional resources and the technological resources during the all lesson. At the end of the lesson, she prepares an assessment on paper. The students have to draw on an empty abacus the beads to represent a given number (for example 91605). When she observes mistakes (for example Yann drew 9165, he forgot to leave an empty rod), she proposes an additional work on the virtual abacus. Indeed on this virtual abacus, it is possible (with a button called “see the number”) to check which number is displayed: hence the students can work autonomously.

**Proposing a training path for teachers**

Rose discovered the existence of the virtual abacus because she participated to a teacher education program. The documentational approach suggests that the design of lessons by groups of teachers, using a given resource, is the best way to lead to the integration of this resource in the teachers’ resource systems, and to a significant and sustainable change in their practice. For these reasons we designed a training path, which provides the structure and the content for a blended training (Riou-Azou, Dhont, Moumin & Poisard 2016).
Figure 4. M@gistère training path: “The Chinese abacus at primary school”

This training path proposes 5 steps, three in presence (1h30 each), and two as distant work. The most important choice is that during step 3 in presence, teams of teachers are formed, and work together to prepare a lesson with the abacus. The lesson is then implemented in classes (step 4); it is presented, discussed and amended during step 5 in presence. The collective documentation work of the teams is central in this training. The training path is available on the national platform M@gistère; the corresponding training can be organised anywhere in the country, by teacher trainers who are not the initial authors of the path, since all the resources to be used are available in the path.

DOCUMENTATION WORK AT HIGHER SECONDARY SCHOOL: “STUDY AND RESEARCH COURSES”.

The study I present here belongs to the project REVEA, meaning “Living Resources for Teaching and Learning”. In this national research project, we follow teachers systems of resources and their evolutions on four subjects: Mathematics, Physics, English and Technology. We followed in particular teachers in a high school during three years. We present here an extract of the case of Gwen, a mathematics teacher who decided in 2016-2017 to set up a “study and research course” entitled “how does a parabolic antenna work”? in her grade 10 class (Gueudet, Lebaud, Otero & Parra submitted).

A study and research course on parabolic antennas, context

Gwen is an experienced teacher: 35 years as upper secondary school teacher, and she participates for many years to groups in the local IREM (Institute for Research on Mathematics Education, proposing thematic groups associating teachers and researchers). She joined this way in September 2016 a group about Study and Research Courses. Study and Research Courses (SRC) have been introduced by Chevallard (2009) as an alternative proposition to the dominant teaching epistemology, where knowledge is presented to the students like works in a museum. At the opposite, the SRC starts by a “generative question”; then the students develop their own inquiry on this question, leading to sub-questions, partial answers etc. The teacher supports this inquiry, but it can develop in different directions, according to the students’ choices. This way the meaning of the mathematics involved appears clearly: they are needed to answer to the question. Many research works have proposed SRC at secondary school or at university, concerning different mathematical topics (e.g. Llanos & Otero 2015, Barachet, Demichel & Noirfalise 2007, Fonseca 2011). Nevertheless, organizing an
SRC in traditional classes is very difficult for teachers, because of the institutional constraints: they have to teach a given content (the official curriculum) in a limited time. Some of the research works on SRC have produced resources to support the implementation in class of SRC by the teachers. This way Gwen has found a booklet proposing SRC, written by a team of an IREM (Bellenoue et al. 2014), and she has decided to set up the SRC called “how does a parabolic antenna work” in her grade 11 class. We have followed this process using the reflective investigation method described above: we set up an interview with Gwen at the beginning of the SRC, observed all the sessions in class, collected all the resources used and produced during the SRC, and had a final interview with Gwen. We analysed all this data by searching for possible operational invariants in Gwen’s declarations, then confronting them to Gwen’s effective activity in order to identify the operational invariants guiding her action and their consequences. We present below examples of these analyses.

Analysing the documentation work of Gwen

The first important element of the documentation work of Gwen is her decision to use the booklet and set up the “Parabolic Antenna” SRC in her class. This decision was guided by several operational invariants (identified through her declaration in the interviews):

- “Giving problems and exercises which are related to a real-life context raises students’ motivation” guided her general intention to give an SRC;
- “The official curriculum of grade 11 must be entirely treated in class” guided her choice of the “Parabolic Antenna” path, which covers several aspects of the program: equations of right lines, of circles, second degree curves, tangents etc.
- Another important operational invariant, for the choice of the “Parabolic Antenna” was that it proposed links between different contents, which are usually taught at different moments of the school year, like the equations of right lines and of circles. This is linked with the operational invariant: “It is interesting to modify the usual organization of the contents”.

Nevertheless, the SRC that Gwen organized in her class is not the SRC planned in the booklet for several reasons.

The major source of modifications is linked with operational invariants, theorems-in-action, that can be formulated as “The available time is limited”, and “The teacher must keep control of the time”. As a
consequence, Gwen modifies the SRC, adding many elements to guide the students – finally the course does not meet the criteria of an SRC.

The booklet already presents a course more guided than an SRC as described by the theory – naturally it is not possible to keep open all the possibilities that the students are likely to investigate, starting from the question: “how does a parabolic antenna work?” For example, the booklet proposes a study of cylindrical mirrors, and how a beam reflects (or not) on such a mirror.

Figure 4. Activity in the booklet: Beams that do not reflect on a cylindrical mirror.

The students are provided with a GeoGebra file, the circle represents the cylindrical mirror of center O. They must construct on GeoGebra the beams coming from the point B₁ that do not reflect on the cylindrical mirror – which means constructing the two tangents to the circle passing through B₁. They observe the equations in the algebra window of GeoGebra, and must compute themselves these equations on paper.

Gwen modifies this activity as follows. The students work by group on paper – not on GeoGebra. They have a sheet with a figure drawn by Gwen on GeoGebra (figure 5). On the figure provided by Gwen, the circle of diameter [OB], which intersections with the initial circle are the two points of contact of the tangents with the mirror circle (indeed the tangents are perpendicular to the radius of this circle) is already present, and the two tangents are constructed as well. Then progressive questions guide the students: find the equations of the two circles, find the coordinates of their intersections D and E, and finally find the equations of the two straight lines (BD) and (BE).
Gwen introduces many modifications of the SRC described in the booklet. Here we only give this example. Our aim is not to criticize Gwen’s courses: she uses a very rich system of resources, articulating the booklet with the class textbook, and other resources found on the Internet. The operational invariants leading her to guide the students much more than in an SRC come from real institutional constraints.

We only wanted to exemplify here how teachers’ knowledge guides their work with resources: the choice of initial resources, and how they modify these resources. On the other hand, the work with the booklet has also modified Gwen’s practices and knowledge. She modified her usual order for the presentation of concepts and developed an operational invariant like “It is interesting to teach the equations of straight lines and the equations of circles together”. She also mentions, in the final interview, how important it is to evidence the meaning of the mathematical concepts taught, and this is directly linked with her experience with the SRC.

CONCLUSIONS

Our conclusions are linked with the two cases presented above, but also with many other cases we studied across different projects using the documentational approach.

The importance of the work of teachers with resources appears clearly. Teachers interact with many resources; the digital resources, and OER in particular intervene in their work in class and out-of-class. The documentational approach is helpful to understand the consequences of these
interactions. It is now used by many colleagues, for all levels of class and for teacher education. It has been used also outside of mathematics: in physics (Alturkmani 2015), chemistry (Hammoud 2012), but also in English (Gruson, Gueudet, Le Hénaff & Lebaud to appear). The comparison between different subjects in terms of interactions between teachers and resources is very informative, and can also contribute to teacher development. In one project were we worked with teachers of English and teachers of mathematics, the teachers of mathematics started asking their students for oral productions (recorded on an MP3), observing that such resources were used by their colleagues in English.

In our different projects, we observe important evolutions linked with digital resources. For example, the collective design by teachers of e-textbooks (Gueudet, Pepin, Sabra & Trouche 2016) is a completely new phenomenon. But we also observe a stability in the practice of the teachers, and this is explained by the documentational approach. The teachers choose indeed, and modify, the resources according to their professional knowledge. Hence even with many resources freely available, the changes in teachers’ practices happen very gradually.

REFERENCES


Hammoud, R. (2012). Le travail collectif des professeurs en chimie comme levier pour la mise en œuvre de démarches d'investigation et le développement des connaissances professionnelles. Contribution au


